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Mathematical model of conflict region in case of three population groups with different priorities

Abstract: The paper proposes a new nonlinear mathematical model, describing in a certain politically conflicting region of a certain state the presence of three population groups with different political priorities. One part of the population (unionists) is politically oriented towards the preservation of the region within the former state, the second part of the population of the region supports the idea of separatism, the separation of the region from the state in order to form a new independent state (separatists), the third part of the population of the region supports the idea of irredentism of the region, that is, secession in order to join another, possibly bordering state (irredentists). A weak (simple majority of the population of the region) and strong (qualified majority of the population of the region) conditions are proposed, which in the legal sense may not have direct consequences, but may determine the aspirations of the majority of the population of the region. The model is described by a nonlinear three-dimensional dynamic system with variable coefficients. Under some assumptions on model parameters, exact analytical solutions were found. Additional conditions were found under which: the region remains within the previous state; possible separation of the region; the irredentism of the region, that is, its accession to another state, is possible.

Keywords: mathematical model, unionists, separatists, irredentist, conflict.

An innovative approach seems to us to study a number of actual social processes, such as the assimilation of languages, globalization, the settlement of political conflicts, the separation of regions, the territorial integrity of states, etc.

From our point of view, the only scientific approach to an adequate quantitative and qualitative description of these complex processes is their mathematical modeling, i.e. the creation of appropriate mathematical models, in the form of multidimensional nonlinear dynamic systems.

We have previously proposed original mathematical models: linguistic globalization, which establishes, within the framework of the model, the possibility of globalization in English [1]; two and three levels of assimilation of languages (peoples) by more common languages [2-4].

We also proposed mathematical models for the settlement of political (not military confrontation) conflicts through the economic cooperation of parts of the populations of the sides with the participation of international organizations and relevant investment funds [5, 6].

Over the past few decades, due to the desire of some political players to redistribute the world map, issues related to the self-determination of nations and the possibility of creating independent states have become relevant.

The paper [7] proposes a general nonlinear mathematical model, which describes the process of the possibility of secession of a particular region from a certain state. The model assumes that only two categories of citizens live in a particular region of a state: the first category, which is a supporter of the center (unionists) and opposes the secession of the region; the second is a supporter of the secession of the region (secessionists, separatists), i.e. its separation from the center, with the aim of forming a new independent state.

We propose a new mathematical model, which is described by the following nonlinear dynamic system

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1(t)u(t) + \big(\beta_1(t) - \beta_2(t)\big)u(t)v(t) + \big(\beta_1(t) - \beta_3(t)\big)u(t)w(t) - \\ \qquad -\gamma_5(t)u(t) - \gamma_3(t)u(t) + \gamma_1(t)v(t) + \gamma_2(t)w(t) \\ \dfrac{dv(t)}{dt} = \alpha_2(t)v(t) + \big(\beta_2(t) - \beta_1(t)\big)u(t)v(t) + \big(\beta_2(t) - \beta_3(t)\big)v(t)w(t) + \\ \qquad +\gamma_3(t)u(t) - \gamma_1(t)v(t) - \gamma_6(t)v(t) + \gamma_4(t)w(t) \\ \dfrac{dw(t)}{dt} = \alpha_3(t)w(t) + \big(\beta_3(t) - \beta_1(t)\big)u(t)w(t) + \big(\beta_3(t) - \beta_2(t)\big)v(t)w(t) + \\ \qquad +\gamma_5(t)u(t) + \gamma_6(t)v(t) - \gamma_2(t)w(t) - \gamma_4(t)w(t) \end{cases} \tag{1}$$

with initial conditions

$$u(0) = u_0,\ v(0) = v_0,\ w(0) = w_0, \tag{2}$$

where

$u(t)$ is the number of supporters of the center (unionists) in the region at time t,

$v(t)$ is the number of supporters of separation from the center in order to create a new independent state (separatists) at the moment t,

$w(t)$ is number of supporters of separation from the center in order to join another state (irredentists) at a time t,

$\alpha_1(t), \alpha_2(t), \alpha_3(t) -$ are demographic factors of the corresponding parts of the population of the region,
$\beta_1(t), \beta_2(t), \beta_3(t) -$ are factors of influence on opponents, in order to attract them to their side (unionism, separatism, irredentism),
$\gamma_1(t), \gamma_2(t) -$ factors of influence of the federal side (the central government of the state) on separatists and irredentists, respectively, in order to attract them to the unionist side,
$\gamma_3(t), \gamma_4(t) -$ factors of influence of external (contributing to separatism) and internal (de facto government) forces on unionists and irredentists, respectively, in order to attract them to the separatist side,
$\gamma_5(t), \gamma_6(t) -$ factors of influence of external interested forces (other state) on unionists and separatists, respectively, in order to attract them to irredentism (reunification with another state),
$[0, T] -$ time interval, model review.

In the model, it is more logical (adequacy of the model) to assume that at the initial moment of time unionists outnumber the total number of separatists and irredentists

$$u_0 > v_0 + w_0. \quad (3)$$

We will consider the weak and strong conditions under which separation of the region is possible, with the aim of creating an independent state (separation of the region) or joining another state (irredentism), which implies the fulfillment of the following inequalities

$$\frac{v(t)}{u(t)+v(t)+w(t)} > \frac{1}{2},\ t > t_1, \quad (4)$$

$$\frac{v(t)}{u(t)+v(t)+w(t)} > \frac{2}{3},\ t > t_2, \quad (5)$$

$$\frac{w(t)}{u(t)+v(t)+w(t)} > \frac{1}{2},\ t > t_3, \quad (6)$$

$$\frac{w(t)}{u(t)+v(t)+w(t)} > \frac{2}{3},\ t > t_4. \quad (7)$$

Weak conditions (4), (6) imply that more than half of the population of the region supports separatism or irredentism, respectively, and strong conditions (5), (7) - more than two thirds of the population of the region (a qualified majority) supports the idea of separating the region and creating a new independent state or reunification with another state.

If none of the inequalities (4)-(7), taking into account (3), then the separation and irredentism of the region is impossible and the conflict region remains part of the previous state.

Consider a special case where there is no influence of forces external to the region (outside the state, as well as the federal center), and unionists, separatists and irredentists only decide among themselves on the choice of the path of political development of the region.

In this case, in the system of equations (1), it is necessary to assume

$$\gamma_i(t) \equiv 0,\ i = \overline{1,6}. \tag{8}$$

Suppose also that the demographic factors of the three parts of the population of the region are zero

$$\alpha_j(t) \equiv 0,\ j = \overline{1,3}. \tag{9}$$

The nonlinear system of differential equations (1) (nonlinear three-dimensional dynamic system), taking

$$\begin{cases} \frac{du(t)}{dt} = \big(\beta_1(t) - \beta_2(t)\big)u(t)v(t) + \big(\beta_1(t) - \beta_3(t)\big)u(t)w(t) \\ \frac{dv(t)}{dt} = \big(\beta_2(t) - \beta_1(t)\big)u(t)v(t) + \big(\beta_2(t) - \beta_3(t)\big)v(t)w(t) \\ \frac{dw(t)}{dt} = \big(\beta_3(t) - \beta_1(t)\big)u(t)w(t) + \big(\beta_3(t) - \beta_2(t)\big)v(t)w(t) \end{cases}. \tag{10}$$

From (10), (2), we get the first integral of a three-dimensional dynamic system

$$u(t) + v(t) + w(t) = u_0 + v_0 + w_0 \equiv p. \tag{11}$$

Consider a special case

$$\beta_1(t) \equiv \beta_2(t) \neq \beta_3(t),\ t\epsilon[0,T]. \tag{12}$$

Considering (12), the second first integral (10), (2) has the following form:

$$v(t) = qu(t),\ q = \frac{v_0}{u_0}. \tag{13}$$

The first two integrals (11), (13) of the dynamic system (10), (2), allow us to find its exact analytical solution

$$\begin{cases} u(t) = \frac{pu_0 e^{p\int_0^t(\beta_1(\tau)-\beta_3(\tau))d\tau}}{w_0+(u_0+v_0)e^{p\int_0^t(\beta_1(\tau)-\beta_3(\tau))d\tau}} \\ v(t) = qu(t) \\ w(t) = p - (q+1)u(t) \end{cases}. \tag{14}$$

Consider a second special case

$$\beta_1(t) \equiv \beta_3(t) \neq \beta_2(t),\ t\epsilon[0,T]. \tag{15}$$

Considering (15), the second first integral (10), (2) has the following form:

$$v(t) = q_1 w(t),\ q_1 = \frac{w_0}{u_0}. \tag{16}$$

The first two integrals (11), (16) of the dynamic system (10), (2), allow us to find its exact analytical solution

$$\begin{cases} u(t) = \frac{pu_0 e^{p\int_0^t (\beta_1(\tau)-\beta_2(\tau))d\tau}}{v_0+(u_0+w_0)e^{p\int_0^t (\beta_1(\tau)-\beta_2(\tau))d\tau}} \\ v(t) = q_1 w(t) \\ v(t) = p-(q_1+1)u(t) \end{cases}. \quad (17)$$

Analysis of the obtained exact analytical solution of the Cauchy problem (10), (2) for a nonlinear three-dimensional dynamic system, under the natural assumption (3) (unionists prevail in the region at the initial moment of time) shows that:

1. In case of execution of inequality system

$$\begin{cases} \beta_1(t) \geq \beta_2(t) \\ \beta_1(t) \geq \beta_3(t) \end{cases}, \quad t\epsilon[0,T] \quad (18)$$

separation or irredentism of the region is impossible and the region in the legal sense remains within the former state.

2. In case of execution of system

$$\begin{cases} \beta_1(t) \equiv \beta_2(t) \\ \beta_3(t) > \beta_1(t) \end{cases}, \quad t\epsilon[0,T] \quad (19)$$

according to (14), regional irredentism is possible (fulfillment of condition (6) or (7)), wherein time t_3 or t_4 is determined from integral relations

$$\int_0^{t_3} \big(\beta_3(\tau)-\beta_1(\tau)\big)d\tau = \frac{ln\frac{u_0+v_0}{w_0}}{p},$$
$$\int_0^{t_4} \big(\beta_3(\tau)-\beta_1(\tau)\big)d\tau = \frac{ln\frac{2(u_0+v_0)}{w_0}}{p}. \quad (20)$$

3. In case of execution of system

$$\begin{cases} \beta_1(t) \equiv \beta_3(t) \\ \beta_2(t) > \beta_1(t) \end{cases}, \quad t\epsilon[0,T] \quad (21)$$

according to (17), it is possible to separate the region (condition (4) or (5)), wherein time t_1 or t_2 is determined from integral relations

$$\int_0^{t_1} \big(\beta_2(\tau)-\beta_1(\tau)\big)d\tau = \frac{ln\frac{u_0+w_0}{v_0}}{p},$$
$$\int_0^{t_2} \big(\beta_2(\tau)-\beta_1(\tau)\big)d\tau = \frac{ln\frac{2(u_0+w_0)}{v_0}}{p}. \quad (22)$$

In conclusion, we would like to note that the proposed mathematical model (1), (2) is common and with variable coefficients of a dynamic system can well describe many conflict regions existing in the world. At the same time, specific conflicts have their own specific sides, characterized by the historical past, the character and mentality of the politically opposing sides (peoples), the geopolitical location and economic potential of the region, the interest of the bordering states, etc., which can be taken into account by the variable parameters of the model. Naturally, with variable coefficients of the mathematical model (1), (2), its analytical solution is impossible, so it is necessary to use computer modeling, using tested computer computing programs.

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